Scientific Project

Abelian varieties with good reduction

The main theme of this proposed research project concerns an algorithmic approach to the theory of abelian varieties and class field theory, subjects of joint interest with David Kohel. Other researchers who might be interested include Yves Aubry, Stephane Ballet, Stephane Louboutin and Serge Vladuts.

Let K be a number field and S a finite set of primes. The set of abelian varieties with good reduction outside S and at worst semistable reduction at primes in S, up to isogeny, form a monoid, generated by the simple abelian varieties with good reduction outside S. In many cases the monoid is finitely generated. A specific example of this phenomenon is the pair $(K, S) = (\mathbf{Q}(\sqrt{6}), \emptyset)$. Indeed, every abelian variety with everywhere good reduction over K is isogenous to a product of copies of the elliptic curve

$$y^2 + \sqrt{6}xy - y = x^3 - (2 + \sqrt{6})x^2.$$

For specific fields K (quadratic or cyclotomic) and ramifying sets, this set of abelian varieties has been provably determined, using methods from class field theory and the theory of finite group schemes. In all known cases the monoid is either zero or is generated by a single simple abelian variety. This poses the natural question to describe a general algorithm for the following problem.

Problem: Given a number field K and set S of primes in K, determine a generator set for the monoid of abelian varieties over K with everywhere good reduction and at worst semistable reduction at primes of S.

A complete solution, either in the form of a deterministic algorithm and its complexity analysis or an implementation suitable for resolving this question in practice for more than a handful of pairs (K, S) is probably inaccessible. Nevertheless, the individual components of an algorithmic solution pose challenging questions of independent interest.

- Compute candidate isogeny classes of simple abelian varieties over K with good reduction outside a finite set of primes S and certain restrictions on the reduction at the primes in S.

- Determine an algorithm for effective lower bounds on the root discriminant of a class field tower under given constraints.
- Compute and classify simple finite commutative groups schemes over number fields K. In particular, classify those of prime order p.

Masters courses or working groups

A common theme to the above investigations is the theory of finite flat group schemes and class field theory. Subject to interest, I propose to teach a masters course on these subjects, or to be involved in a working group on group schemes, class field theory or modular curves.