

# Inverse problems and related fields

IP&RF17

- 5<sup>ième</sup> édition -

Jeudi 23 Novembre - Vendredi 24 Novembre 2017

FRUMAM, AMU, Marseille

web site : <https://iprf-ma17.sciencesconf.org/>





## Présentation

Dans le cadre du groupe de travail GOMS deux journées tournant autour des problèmes inverses sont organisées à Marseille du Jeudi 24 Novembre au 25 Novembre 2016 dans les locaux de la FRUMAM de l'Université d'Aix-Marseille. Ces journées sont organisées par Michel Cristofol (I2M) et Eric Soccorsi (CPT). Cette année ces journées sont soutenues par Aix marseille Université, l'I2M, l'équipe d'Analyse appliquée de l'I2M, le CPT, le GDR CATIA, le GDR DYNQUA, le Labex Archimède, la FRUMAM et le groupe de travail GOMS.

Ces journées présenteront des communications sur des avancées récentes dans des domaines aux interfaces entre les problèmes inverses, le contrôle et l'analyse des EDP. Elles s'adressent à tous les chercheurs intéressés ou curieux et aux jeunes chercheurs en formation (doctorants et postdoctorants).

Ces journées se déroulent dans les locaux de la Frumam, 3 Place Victor Hugo, Marseille (site Web : [www.frumam.cnrs-mrs.fr](http://www.frumam.cnrs-mrs.fr)).

## Conférenciers

Paul BARBONE (Université de Boston, USA)

Karine BEAUCHARD (ENS Rennes, France)

Piermarco CANNARSA (Université de Roma, Italie)

Anna DOUBOVA (Université de Séville, Espagne)

Moncef MAHJOUB (LAMSIN, Tunisie)

Kaori NAGATOU PLUM (Karlsruher Institut für Technologie (KIT), Allemagne)

Eva SINCICH (Université de Trieste, Italie)

Masahiro YAMAMOTO (Université de Tokyo, Japon)

## Programme

### Jeudi 23 Novembre

- 12h00 – 14h00** **Accueil des participants.** Le déjeuner est pris traditionnellement près du site de la Frumam. Rejoignez nous à la Frumam si vous êtes arrivés sur Marseille.
- 14h00 – 15h00** **Moncef MAHJOUB**  
Modelling and numerical simulation in cardiac electrophysiology : reduced order modelling and inverse problem.
- 15h00 – 16h00** **Kaori NAGATOU PLUM**  
An approach to computer-assisted existence proofs for nonlinear space-time fractional parabolic problems.
- 16h00 – 16h20** **Pause Café- Discussions**
- 16h20 – 17h20** **Piermarco CANNARSA**  
Control of degenerate parabolic operators in low space dimension : old and new.
- 20h00** **Diner** au Pavillon flottant : "Les 2 pointus"

### Vendredi 24 Novembre

- 9h00 – 10h00** **Karine BEAUCHARD**  
Quadratic obstructions to small-time local controllability for scalar-input differential systems
- 10h – 10h20** **Pause Café**
- 10h20 – 11h20** **Paul BARBONE**  
TBA.
- 11h20 – 12h20** **Eva SINCICH**  
Lipschitz stability for a piecewise linear Schrödinger potential from local Cauchy data.
- 12h20 – 14h00** **Déjeuner**
- 14h00 – 15h00** **Anna DOUBOVA**  
Numerical approximations of geometric inverse problems of some PDEs.
- 15h00 – 16h00** **Masahiro YAMAMOTO**  
TBA.
- 16h00 – 16h20** **Pause Café- Discussions**

## Résumés des exposés

**Paul BARBONE**

**Title :** *TBA.*

**Abstract :**

**Karine BEAUCHARD**

**Title :** *Quadratic obstructions to small-time local controllability for scalar-input differential systems.*

**Abstract :** We consider nonlinear scalar-input differential control systems in the vicinity of an equilibrium. When the linearized system at the equilibrium is controllable, the nonlinear system is smoothly small-time locally controllable, i.e., whatever  $m > 0$  and  $T > 0$ , the state can reach a whole neighborhood of the equilibrium at time  $T$  with controls arbitrary small in  $C^m$ -norm. When the linearized system is not controllable, we prove that small-time local controllability cannot be recovered from the quadratic expansion and that the following quadratic alternative holds. Either the state is constrained to live within a smooth strict invariant manifold, up to a cubic residual, or the quadratic order adds a signed drift in the evolution with respect to this manifold. In the second case, the quadratic drift holds along an explicit Lie bracket of length  $(2k + 1)$ , it is quantified in terms of an  $H^{-k}$ -norm of the control, it holds for controls small in  $W^{2k, \infty}$ -norm. These spaces are optimal for general nonlinear systems and are slightly improved in the particular case of control-affine systems. Unlike other works based on Lie-series formalism, our proof is based on an explicit computation of the quadratic terms by means of appropriate transformations. In particular, it does not require that the vector fields defining the dynamic are smooth. We prove that  $C^3$  regularity is sufficient for our alternative to hold. This work underlines the importance of the norm used in the smallness assumption on the control : depending on this choice of functional setting, the same system may or may not be small-time locally controllable, even though the state lives within a finite dimensional space. .

**Piermarco CANNARSA**

**Title :** *Control of degenerate parabolic operators in low space dimension : old and new.*

**Abstract :** The null controllability properties of degenerate parabolic operators in low space dimension, via either boundary or locally distributed controls, are by now fairly well understood. They will be the main topic of this talk, in which special attention will be devoted to estimates for the blow-up of the cost of control as degeneracy approaches the controllability threshold.

**Anna DOUBOVA**

**Title :** *Numerical approximations of geometric inverse problems of some PDEs.*

**Abstract :** We will focus our talk on the numerical approximations of geometric inverse problem for some PDEs motivated by Elastography. We present several recent results and open questions concerning the numerical reconstruction of the unknown domain where the equations evolve. In the numerical experiments, we solve an appropriate optimization problems. Two different numerical techniques will be proposed. Firstly, the FEM for the numerical solution of the PDE's, performed with FreeFem++ . The routines on the ff-NLOpt package, that provide an interface to a free/open-source library for nonlinear optimization, are also required. On the other hand, we will consider the numerical approximation based on the method of fundamental solutions. It deals with a meshless method. We present some numerical results in the 2D and 3D cases. The first part is joint work with E. Fernandez-Cara and the second part is joint work with P. Carvalho, E. Fernandez-Cara and J. Rocha.

**Moncef MAHJOUB**

**Title :** *Modelling and numerical simulation in cardiac electrophysiology : reduced order modelling and inverse problem.*

**Abstract :** The electric wave in the heart is governed by a system of reaction-diffusion partial differential equations called the bidomain model. This system is coupled nonlinearly to an ordinary differential equations (ODEs) modeling the cellular membrane dynamics. The bidomain model is widely used in cardiac electrophysiology simulation. This mathematical model takes into account the electrical properties of the cardiac muscle. Different numerical methods have been used for solving the bidomain model. Finite element method, finite difference method and finite volume method. All these methods lead to a large linear system to solve, especially when using implicit schemes. The numerical cost in this case becomes very important when we are interested in solving inverse problems. The computational cost of solving the bidomain problem becomes very important when we are interested in solving inverse problem. Thus reducing the computational cost of the forward problem is a challenging issue. One of the most popular approaches used in model reduction is the *Proper Orthogonal Decomposition* method. This method was initially introduced for analyzing multidimensional data.

In this work we present a reduced order approach based on POD method for the computation of the electrical activity of the heart. We stability result of the POD method based on an a priori error estimate. This theoretical result shows that we can control the gap between the full finite element solution and the POD solution of the bidomain equation by controlling the gap between the finite element solution and its projection on the used POD basis. We also show that the POD method could be used for different strategies of solving the bidomain model. It could be used for a fully coupled scheme or by using a time splitting schemes. The numerical results show that it is stable in both cases. In order to evaluate the usefulness of this approach in parameter estimation problem, we build a POD basis using the original parameters of the ionic model and we computed the  $L^2$  relative error between the finite elements solution and the reduced order solution for different parameters. Also we conclude that in case of parameter estimation framework it is recommended to use the POD in order to estimate  $\tau_{close}$ ,  $\tau_{open}$  and  $\tau_{out}$  introduced in the formula of the ionic current model. But to estimate the parameter  $\tau_{in}$ , the data from which the POD basis is computed should be sufficiently rich in order to maintain a good accuracy of the results.

In the second part of this work, we will interest to established a theoretical stability estimates for the parameter identification  $\tau_{in}$  to which the solution is the most sensitive. We established a new Carleman inequality for a reaction diffusion equation coupled to an ordinary differential equation. The Carleman inequality that we established for the ODE was fundamental in order to prove the global Carleman estimate for non linear parabolic equation coupled with an ordinary differential equation and solving the parameter stability problem.

**Kaori NAGATOU PLUM**

**Title :** *An approach to computer-assisted existence proofs for nonlinear space-time fractional parabolic problems.*

**Abstract :**

**Eva SINCICH**

**Title :** *Lipschitz stability for the electrostatic inverse boundary value problem with piecewise linear conductivities.*

**Abstract :** We consider the electrostatic inverse boundary value problem also known as electrical impedance tomography (EIT) for the case where the conductivity is a piecewise linear function on a domain  $\Omega \subset \mathbb{R}^n$  and we show that a Lipschitz stability estimate for the conductivity in terms of the local Dirichlet-to-Neumann map holds true. This is based on a joint work with G. Alessandrini, M.V. de Hoop and R. Gaburro.

**Title :**

**Abstract :**

**Masahiro YAMAMOTO**

**Title :** *TBA .*

**Abstract :**

## Emails des orateurs et des organisateurs

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